SMOOTHING OF STATIONARY RANDOM SIGNAL IN CONTINUOUS FLOW MIXER WITH GAMMA-DISTRIBUTION OF RESIDENCE TIMES*

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The paper is focused on a theoretical analysis of the function of continuous flow mixer with the so-called gamma-distribution of fluid residence times, used as a linear filter smoothing undesirable fluctuations of input properties. A relation is derived expressing the degree of smoothing of the signal passing through the system, as a function of statistical parameters of this signal and of gamma-distribution of fluid residence times in the mixer. The analysis of this relation leads to conclusions concerning the prediction of the operation of smoothing mixers or the design of their basic parameters.

In chemical industry and in some related branches we may often observe that intense qualities (*e.g.* concentration of components or temperature) of products flow in continuous technological processes are subject to random fluctuations about a certain (constant) mean value. These random fluctuations are due to the influence of a number of uncontrollable factors and are often superimposed by periodical changes caused by regular repetition of some technological operations.

As these fluctuations have a negative influence on the production (causing variations of its quality) it is necessary to limit their range to an acceptable value. The simplest means to achieve this end is the introduction of a continuous flow mixer with a sufficient capacity into the flow. There it may operate as a linear capacity filter whose transfer function is determined by the residence times distribution of fluid. In our last paper[1] we investigated the problems of such application of a continuous flow mixer for the general case, in which the distribution of fluid residence times in the mixer is a nonstationary random function of time. Now we shall focus on the commonly considered case – *i.e.* the distribution of fluid residence times will be considered as a deterministic function of time which is invariant with regard to the shift along the time axis.

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In spite of the attempts of many authors[2-21], the problems of elimination of undesirable fluctuations in product flows still lack a complex treatment. Considerable attention has been paid to the smoothing of periodical (especially harmonic and rectangular) concentration fluctuations in an ideal continuous flow mixer[5-9], in the same mixer with a bypassing or recirculation stream[9-13] and in the cascade composed of ideal continuous flow mixers[7,14]. The results of these studies prove that the degree of smoothing increases along with the growing mean residence time of fluid in the mixer and with growing fluctuation frequency. In systems as mixer — bypass or mixer — recirculation stream full smoothing may be attained if the parameters are chosen optimally.

In studies dealing with the smoothing of random fluctuations such phenomena have been treated[9,10,15-21] as a stationary random function of time with the autocorrelation function in the form

$$R_x(\tau) = \sigma_x^2 \exp(-A|\tau|) . \tag{1}$$

Little notice was taken[20,21] of stationary random fluctuations with a prominent (dominant) harmonic component, whose autocorrelation function may be expressed in the form

$$R_x(\tau) = \sigma_x^2 \exp(-A|\tau|) \cos B\tau . \tag{2}$$

In many cases[2-4,15,17,18] again an ideal continuous flow mixer has been considered as a model equipment. In other cases[9,10,19,20,21] the investigation has been focused on this mixer combined with a bypass or recirculation flow. Some attention has also been given to parallel[20,21] and serial[15,20,21] combinations of some ideal mixers. The results of studies dealing with smoothing of random fluctuations show that its degree increases with increasing mean residence time and with increasing values of the parameters A and B of the autocorrelation functions (*1*) and (*2*). It has been proved theoretically[19-21] that in certain cases better smoothing that in the mixer alone attacan beined by the combination of mixer and bypass. The above survey of literature shows that in modelling continuous flow smoothing mixers only rather simplified ideas about the character of liquid flow in the equipment have been considered. These notions cannot with sufficient accuracy and flexibility express the non-ideal nature of fluid flow in mixers used in technological practice. Fluctuations of mixer input properties have been considered either periodical or random with the autocorrelation function according to (*1*). Only very small attention has been paid to the case most interesting in practice, namely to the combination of random fluctuations with periodical oscillations; its autocorrelation function has been described by (*2*).

This study attempts to fill out at least some of the mentioned blank spaces in the research of smoothing mixers. The fluctuations of mixer input properties are conceived as a stationary random process with an autocorrelation function according to (*2*). The so-called gamma-distribution, *e.g.*[22] is used to describe the distribution of fluid residence times in the mixer. Its flexibility and suitability for such description is apparent from literature[23-27] and from our previous studies[28-30]. The gamma-distribution shall be used in the same form as in the latter works

$$f(t) = \frac{\varkappa\beta}{\Gamma(\beta)} \exp(-\varkappa\beta t)(\varkappa\beta t)^{\beta-1} . \tag{3}$$

The $\varkappa$ value represents[28,29] the reciprocal value of the mean residence time of the fluid in the mixer; the β value quantitatively describes the intensity of mixing. In the limiting case of ideal mixer $\beta = 1$; in the case of piston flow $\beta \to \infty$.

THEORETICAL

Let us consider a continuous flow mixer with one inlet and outlet stream under stationary regime (constant hold-up of fluid, input and flow regime). Let us further assume that the influence of mixer's own fluctuations may be neglected within the range of sufficient accuracy. The time variation of the investigated property (*e.g.* concentration of some component) in the mixer inflow shall be the stationary random process $X(t)$ with the variance σ_x^2 and the autocorrelation function $R_x(\tau)$ according to (*2*). This process will henceforth be described as mixer input. The smoothed fluctuations in the mixer output are again a stationary random process[22,31] denoted $Y(t)$ which is called mixer response. Its variance is denoted σ_y^2 and its autocorrelation function $R_y(\tau)$.

Let us now define the smoothing effect of the mixer Q in the usual way[9,10,15–21], as the ratio of mixer response and input

$$Q = \sigma_y^2/\sigma_x^2 \,. \tag{4}$$

It is immediately evident that if $Q = 0$, the signal has been smoothed completely and if $Q = 1$, the signal is not smoothed at all.

To derive the expression of mixer smoothing effect Q, the well known relation[22,31] may be used

$$R_y(\tau) = \int_0^\infty f(u) \int_0^\infty f(v)\, R_x(\tau + u - v)\, \mathrm{d}u\, \mathrm{d}v \,. \tag{5}$$

This holds for a linear dynamic system with constant parameters described by the impulse response $f(t)$, into whose entrance a stationary random process $X(t)$ is introduced. The validity of (*5*) is conditioned by the fulfillment of the assumption about the stochastic independence of functions $f(t)$ and $X(t)$, *i.e.* that the character of the liquid flow inside the mixer is not influenced by the value of the investigated input property of the entering flow. In case of continuous flow mixers this assumption is usually fulfilled with sufficient accuracy. The response to unit impulse $f(t)$ of the mixer coincides, if the mentioned assumptions are valid, with the probability density of fluid residence times in the mixer.

After introducing the normalized autocorrelation function $r_x(\tau)$ of the mixer input

$$r_x(\tau) = R_x(\tau)/R_x(0) \tag{6}$$

and with respect to relations[22,21]

$$R_x(0) = \sigma_x^2 \,, \quad R_y(0) = \sigma_y^2 \,, \tag{7}$$

we shall obtain from (5)

$$Q = \int_0^\infty f(u) \int_0^\infty f(v)\, r_x(u - v)\, du\, dv\,. \quad (8)$$

After substitution from (2) and (3) we shall get

$$Q = \frac{(\varkappa\beta)^2}{\Gamma^2(\beta)} \int_0^\infty u^{\beta-1} \exp^{(-\varkappa\beta u)} \int_0^\infty v^{\beta-1} \exp^{(-\varkappa\beta v)} \exp^{(-A|u-v|)} \cos B(u - v)\, du\, dv\,. \quad (9)$$

Solving the integrals, Eq. (9) yields the relation for Q in the form[32]

$$Q = \frac{\Gamma(2\beta)}{2^{2\beta}\beta\Gamma^2(\beta)} \left[F\left(1, 2\beta; \beta + 1; \frac{\varkappa\beta - A + iB}{2\varkappa\beta}\right) + \right. \\ \left. + F\left(1, 2\beta; \beta + 1; \frac{\varkappa\beta - A - iB}{2\varkappa\beta}\right)\right], \quad (10)$$

where the symbol $F(.)$ denotes the so-called hypergeometric function[22] of the respective arguments. After writing the hypergeometric function in the form of infinite series[22] and after rearrangements the final relation is obtained

$$Q = \frac{2\varkappa\beta(\beta + 1/2)}{\sqrt{(\pi)}\,\Gamma(1 - \beta)\,[(A + \varkappa\beta)^2 + B^2]} \sum_{k=0}^{\infty} \varrho^k [(A + \varkappa\beta)\cos(k\varphi) + B\sin(k\varphi)]\,. \\ \cdot \frac{\Gamma(k + 1 - \beta)}{\Gamma(k + 1 + \beta)}, \quad (11)$$

where parameters ϱ and φ are defined by the relations

$$\varrho = \left|\frac{(A - \varkappa\beta)^2 + B^2}{(A + \varkappa\beta)^2 + B^2}\right|^{1/2} \quad (12)$$

and

$$\varphi = \begin{cases} \pi, \quad \text{for} \quad A^2 + B^2 < (\varkappa\beta)^2 \\ 2\,\mathrm{arc\,tg}\,[2\varkappa\beta B/\{(A^2 + B^2 - \varkappa^2\beta^2) + [(A^2 + B^2 - \varkappa^2\beta^2)^2 + (2\varkappa\beta B)^2]^{1/2}\}]\,, \end{cases} \quad (13)$$

in the remaining cases.

The infinite series in Eq. (11) is convergent[22], because for the arguments of hypergeometric functions in Eq. (10) it holds

$$\left|\frac{\varkappa\beta - A - ib}{2\varkappa\beta}\right| = \left|\frac{\varkappa\beta - A + ib}{2\varkappa\beta}\right| \leqq 1\,. \quad (14)$$

Eqs (*11*) to (*13*) then enable us to calculate the smoothing effect of the mixer with residence time distribution in the form of gamma-distribution (*3*) on condition that the stationary random signal with the autocorrelation function given by (*2*) is being introduced into its entry.

DISCUSSION

Analysis and discussion of relation (*11*) may be facilitated by reducing the number of variables by introducing the dimensionless quantities

$$p \equiv A/\varkappa \quad \text{and} \quad q \equiv B/\varkappa \,. \tag{15}$$

After substituting p and q into Eqs (*11*) to (*13*), the smoothing effect of the mixer is defined by

$$Q = \frac{2\beta\Gamma(\beta + 1/2)}{\sqrt{(\pi)}\,\Gamma(1 - \beta)\,[(p + \beta)^2 + q^2]} \sum_{k=0}^{\infty} \varrho^k [(p + \beta)\cos(k\varphi) + q \sin(k\varphi)] \,. \frac{\Gamma(k + 1 - \beta)}{\Gamma(k + 1 + \beta)}\,, \tag{16}$$

$$\varrho = \left[\frac{(p - \beta)^2 + q^2}{(p + \beta)^2 + q^2}\right]^{1/2}, \tag{17}$$

$$\varphi = \begin{cases} \pi & \text{for} \quad p^2 + q^2 < \beta^2 \\ 2 \operatorname{arc\,tg} [2\beta q / \{(p^2 + q^2 - \beta^2) + [(p^2 + q^2 - \beta^2)^2 + (2\beta q)^2]^{1/2}\}] \end{cases} \tag{18}$$

in the remaining cases.

Before starting to discuss the dependence of the smoothing effect Q on the individual variables, we shall introduce some special and limiting forms of Eq. (*16*):

A) For $q = 0$ and $p > 0$. This case describes a situation, when the input signal of the mixer does not contain a prominent harmonic component, *i.e.* when the parameter B of the autocorrelation function (*2*) equals zero (see also (*15*)). Autocorrelation function of the mixer input is then described by the commonly used relation (*1*). For the smoothing effect a simplified relation derived from (*16*) shall hold

$$Q = \frac{2\beta\Gamma(\beta + 1/2)}{\sqrt{(\pi)}\,(p + \beta)\,\Gamma(1 - \beta)} \sum_{k=0}^{\infty} \left(\frac{p - \beta}{p + \beta}\right)^k \frac{\Gamma(k + 1 - \beta)}{\Gamma(k + 1 + \beta)}\,, \tag{19}$$

which for integer values takes the form of a finite sum describing the smoothing effect of the cascade of ideal mixers[15]. For an ideal mixer (*i.e.* for $\beta = 1$), Eq. (*16*) will be transformed in the well-known relation[10,15-21]

$$Q = \frac{1}{1 + p} = \frac{\varkappa}{\varkappa + a}. \tag{20}$$

B) For $p = 0$ and $q > 0$. Here the parameter A of the autocorrelation function of the input signal of the mixer equals zero (see (*2*) and (*15*)). Hence the input signal may be represented by the harmonic function of time with angular frequency $\omega = B$. If $p = 0$, Eq. (*16*) shall appear in the form

$$Q = \frac{2\beta\Gamma(\beta + 1/2)}{\sqrt{(\pi)}\,\Gamma(1 - \beta)(\beta^2 + q^2)} \sum_{k=0}^{\infty} (\beta \cos(k\varphi) + q \sin(k\varphi)) \frac{\Gamma(k + 1 - \beta)}{\Gamma(k + 1 + \beta)}, \tag{21}$$

which has already been proved[32] equivalent to

$$Q = [\beta^2/(\beta^2 + q^2)]^\beta. \tag{22}$$

This relation may be easily obtained as the square of the absolute value of the Fourier transformation of gamma-distribution (*3*).

C) For $\beta = 1$. This case describes the smoothing of stationary random signal with autocorrelation function (*2*) in the ideal continuous flow mixer with the average residence time of fluid $\bar{t} = 1/\varkappa$. From (*16*) we obtain for $\beta = 1$

$$Q = (p + 1)/[(p + 1)^2 + q^2]. \tag{23}$$

The special form of this relation is Eq. (*20*) (see *A*). Eq. (*23*) may also be easily derived after substituting the probability density of residence times in the ideal continuous flow mixer

$$f_i(t) = \varkappa \exp^{(-\varkappa t)}, \quad (\beta = 1) \tag{24}$$

directly into Eq. (*8*) and after solving the respective integrals.

D) For $\beta \to \infty$. As it was shown after Eq. (*3*), the gamma-distribution (*3*) transforms in the limiting case $\beta \to \infty$ into the form of the probability density of residence times of fluid in the equipment with piston flow

$$f_p(t) = \delta(t - 1/\varkappa), \quad (\beta \to \infty). \tag{25}$$

From Eqs (*17*) and (*18*) it is evident that

$$\lim_{\beta \to \infty} \varrho = 1 \tag{26a}$$

and

$$\lim_{\beta \to \infty} \varphi = \pi \,. \tag{26b}$$

After substitution from Eqs (*26*) into (*16*) we shall obtain

$$Q = \frac{2\Gamma(\beta + 1/2)}{\sqrt{(\pi)}\Gamma(1 - \beta)} \sum_{k=0}^{\infty} \frac{\Gamma(k + 1 - \beta)}{\Gamma(k + 1 + \beta)} (-1)^k \,. \tag{27}$$

The infinite series expresses (apart from a constant factor) the hypergeometric function[22] $F(1, 1 - \beta; 1 + \beta; -1)$, for which

$$F(1, 1 - \beta; 1 + \beta; -1) = \frac{\sqrt{\pi}}{2} \frac{\Gamma(\beta + 1)}{\Gamma(\beta + 1/2)} \,. \tag{28}$$

By the connection of Eqs (*27*) and (*28*) and after slight rearrangement we obtain

$$\lim_{\beta \to \infty} Q = \lim_{\beta \to \infty} \frac{\beta(p + \beta)}{(p + \beta)^2 + q^2} = 1 \,, \tag{29}$$

hence the expected result for an equipment with the piston flow of fluid.

E) For $p = 0$ and simultaneously $q = 0$ $[A \neq 0, B \neq 0]$. It follows from (*15*) that this case describes the limiting situation $\varkappa \to \infty$, *i.e.* the mean value of fluid residence time in the mixer approaches zero. From (*17*) and (*18*) it is evident that in this case again the equalities (*26*) hold. Using relations (*27*) to (*29*) we may easily obtain the result

$$Q = 1 \,, \quad (p, q \mp 0) \,, \tag{30}$$

which has been expected for the mixer with the zero mean residence time of fluid.

F) For $p \to \infty$ and/or $q \to \infty$. From Eqs (*17*) and (*18*) the validity of the following relations is evident

$$\lim_{p \to \infty} \varrho = \lim_{q \to \infty} \varrho = \lim_{p,q \to \infty} \varrho = 1 \,, \tag{31}$$

$$\lim_{p \to \infty} \varphi = \lim_{q \to \infty} \varphi = \lim_{p,q \to \infty} \varphi = 0 \,. \tag{32}$$

The limit for $p \to \infty$ (and for $\varkappa \neq 0$) describes (see (*15*)) the situation in which the A value of the mixer input grows to infinity, *i.e.* the signal has features of the so-called white noise[22,31]. The limit for $q \to \infty$ (and for $\varkappa \neq 0$) describes the situation when the value of the parameter B of the autocorrelation function of the input, *i.e.* the value of angular frequency of the dominant harmonic component of this signal, grows to infinity. The limit for $p \to \infty$ and simultaneously for $q \to \infty$ describes the case, when the $\varkappa$ value approaches zero, *i.e.* when the mean residence time of fluid in the mixer grows to infinity.

It can be easily shown that if Eqs (*31*) and (*32*) are valid, the following relation holds

$$\lim_{p \to \infty} Q = \lim_{q \to \infty} Q = \lim_{p,q \to \infty} Q = 0\,, \tag{33}$$

i.e. the smoothing of the signal in the mixer is perfect in all the three cases.

The dependence of the smoothing effect of the mixer Q on the variables p, q, and β is shown in Figs 1 to 6. Fig. 1 depicts in isometric projection the area $Q = Q(p, q, \beta = 1)$, *i.e.* the simultaneous dependence of the smoothing effect of the ideal continuous flow mixer on the variables p and q. Figs 2 to 6 illustrate particular dependencies of Q on individual variables.

Figs 1, 2, and 4 demonstrate that the dependence of the smoothing effect Q on the variable p (at q = const. and β = const.) acquires two qualitatively different forms depending on the values of q and β. If the value of the variable q (at β = const.)

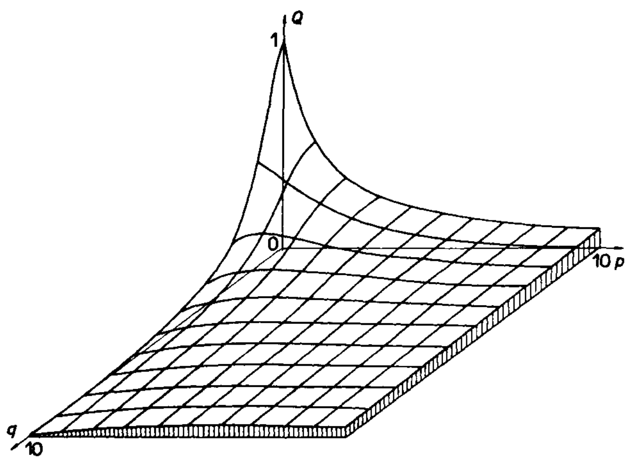

FIG. 1

Simultaneous dependence of smoothing effect Q of ideal mixer on variables p and q (according to Eq. (*16*) for $\beta = 1$)

is lower than a certain critical value q_c, the value of smoothing effect Q decreases monotonously with the growing value of the variable p, *i.e.* when the value of parameter A of the autocorrelation function of the mixer input increases. Consequently,

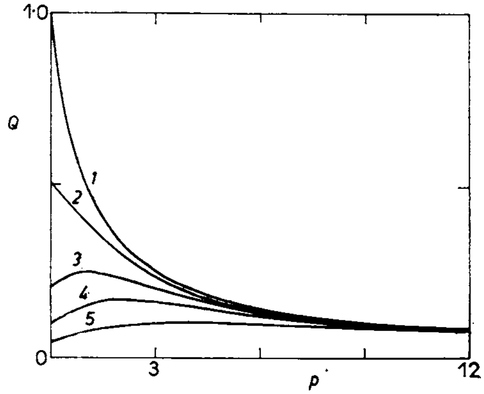

FIG. 2

Dependence of smoothing effect Q of ideal mixer ($\beta = 1$) on variable p for various values of parameter q *1*: $q = 0$; *2*: $q = 1$; *3*: $q = 2$; *4*: $q = 3$; *5*: $q = 5$

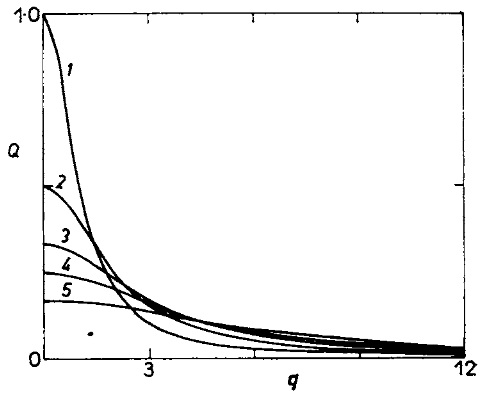

FIG. 3

Dependence of smoothing effect Q of ideal mixer ($\beta = 1$) on variable q for various p values *1*: $p = 0$; *2*: $p = 1$; *3*: $p = 2$; *4*: $p = 3$; *5*: $p = 5$

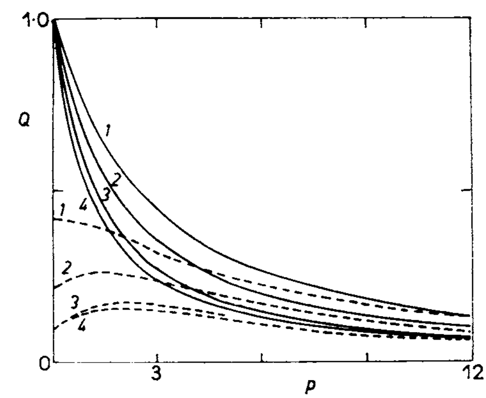

FIG. 4

Dependence of smoothing effect Q on variable p for various values of β and q ——— $q = 0$; ······· $q = 3$. *1*: $\beta = 10$; *2*: $\beta = 5$; *3*: $\beta = 2$; *4*: $\beta = 1$

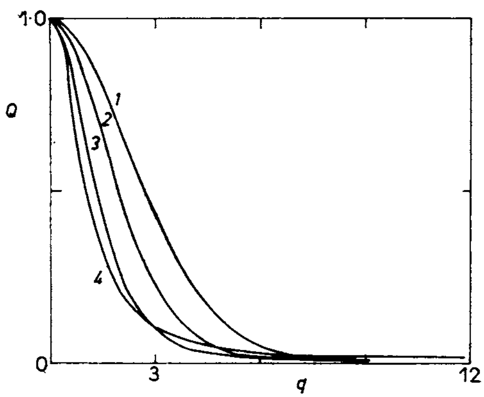

FIG. 5

Dependence of smoothing effect Q on variable q for $p = 0$ and various values of β. *1*: $\beta = 10$; *2*: $\beta = 5$; *3*: $\beta = 2$; *4*: $\beta = 1$

the smoothing of signals autocorrelated over shorter time intervals (in which the dependence between two values of the signal in different moment rapidly decreases with the increase of the difference between these moments) is better than smoothing of the signal correlated over longer time intervals. If $q > q_c$, the dependences of Q on the variable p are characterized by rather flat local peaks. For an ideal continuous flow mixer ($\beta = 1$) the q_c value and p_m values of the variable p at which Q amounts to maximum, may be easily determined from Eq. (*23*):

$$\left.\begin{array}{l} q_c = 1 \\ p_m = q - 1 \end{array}\right\} \quad \text{for} \quad \beta = 1 \quad \text{and} \quad q \geqq q_c \,. \qquad \begin{array}{r} (34a) \\ (34b) \end{array}$$

For the remaining values of β, the values of q_c and p_m can be determined from (*16*) by numerical methods.

The appearance of the maxima just described (*i.e.* the regions in which the smoothing capacity of the mixer is limited) may be explained by a kind of interference between the random component of the input (characterized by A) and the dominant harmonic component of this signal (described by B), which occurs at certain values of the mean residence time of fluid in the mixer.

The dependence of Q on the variable q (at p = const., β = const.) is always a monotonously decreasing function of this variable – see Figs 1, 2, and 5. The smoothing of the signal passing through the mixer may thus become the more perfect, the higher the frequency of its dominant harmonic component. Figs 2 and 4 may further illustrate the improvement of the smoothing capacity of the mixer in case of input signals with a pronounced harmonic component ($q > 0$), in contrast to signals lacking such component ($q = 0$). This effect is especially conspicuous

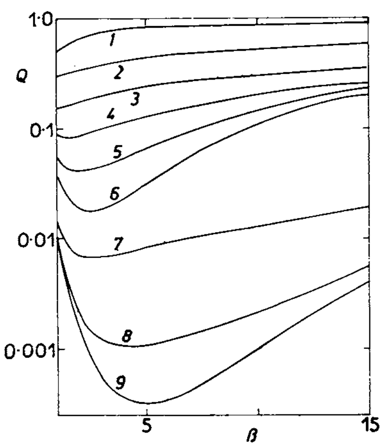

FIG. 6

Dependence of smoothing effect Q on parameter β at various p and q values. *1*: $q = 1$, $p = 0$; *2*: $q = 1$, $p = 2$; *3*: $q = 1$, $p = 10$; *4*: $q = 5$, $p = 2$; *5*: $q = 0.5$, $p = 0.5$; *6*: $q = 5$, $p = 0$; *7*: $q = 10$, $p = 0.5$; *8*: $q = 10$, $p = 0.05$; *9*: $q = 10$, $p = 0$

in the area of low values of p (approx. $p \leqq 6$), namely for the input signal correlated over longer time intervals.

The dependence of Q on the parameter $\varkappa$ of the gamma-distribution (*3*) is not explicitely expressed in any of the figures. But definitions (*15*) imply the inverse proportionality between p and $\varkappa$ or q and $\varkappa$ (A = const., B = const.). Hence, in the coordinate plane p, q (see Fig. 1) the changes of $\varkappa$ correspond to the simultaneous changes of p and q: the decrease of $\varkappa$ value is thus represented (A = const., B = = const.) by the movement along the half-line from the origin of the coordinate system p, q. Fig. 1 (and Figs 2 to 5, in case $p = 0$ or $q = 0$) make evident that the value of smoothing effect Q decreases (the smoothing of the signal improves) with the decreasing value of $\varkappa$, *i.e.* with the increasing value of the mean residence time of fluid in the mixer.

The dependence of Q on the parameter β of the gamma-distribution (*3*) is depicted in Figs 4, 5, and 6. Similarly as in the case of the dependence of Q on the variable p, the dependence of the smoothing effect Q on β (when p = const. and q = const.) shows two different forms, either a monotonous growth of Q with the increase of the β value or the existence of (rather shallow) local minima in its course. These minima are more conspicuous at low p values, *i.e.* in the cases when the input signal approaches to the harmonic function of time.

For the purely harmonic input signal ($p = 0$) Eq. (*22*) – introduced for the integer values of β, *i.e.* for the cascade of ideal mixers, already by Kramers and Alberda[14] – a simple relation may be derived for the optimum value β_0, at which Q reaches minimum value

$$\beta_0 = cq \quad \text{and} \quad \beta_0 \geqq 1\,, \tag{35}$$

where $c \doteq 0{\cdot}50498$ is a constant. Formally identical equation for the optimum number of ideal mixers and the same value of c has been given by Hiby, Melin and Tsuge[7] who used Kramers' and Alberda's results. For the non-zero values of p, the value of β_0 may be determined numerically from Eq. (*16*) (if it exists at the given p and q – see Fig. 6).

These results show that for harmonic input signals or those approaching the harmonic function the ideal continuous flow mixer may not represent (at higher values of signal frequency) the most effective smoothing equipment.

CONCLUSION

Resulting relations (*11*) and (*16*) for the smoothing function of the continuous flow mixer are derived on condition that the input signal of the mixer is a stationary random function of time with the autocorrelation function according to (*2*) and that the distribution of residence times of fluid in the mixer can be described by gamma-

-distribution (*3*). These chosen forms of autocorrelation function of the input signal and of the distribution of fluid residence times are sufficiently flexible and universal, so that an extensive set of causes often met in practice may be described by means of Eqs (*11*) and (*16*).

The results of the theoretical analysis of (*16*) agree with the physical ideas of the problem, or with some results of previously published works. The assumptions of the stationary property of input signal and the deterministic character of the distribution of fluid residence times in the mixer set no serious restrictions to the practical application of (*11*) and (*16*). The results may be easily extended to the case of mixers with more inflows and outflows.

Eqs (*11*) and (*16*) may then be used for the assessment of the operation of continuous flow mixers, or for the suggestion of their parameters. In doing so, the results of the preceding paragraph should be taken into account, especially the possible occurence of such combination of values of independent variables, due to which the smoothing effect of the mixer may attain locally extreme values.

A certain – though nowadays hardly serious – drawback of the resulting relations is their complicated form. Yet if the algorithm is appropriately chosen, calculations of the smoothing effect may be carried out even by means of an ordinary programmable calculator.

LIST OF SYMBOLS

A	parameter of autocorrelation function(2) (T^{-1})
B	parameter of autocorrelation function(2) (T^{-1})
$f(t)$	probability density of residence times (T^{-1})
$F(a, b; c; d)$	hypergeometric function
i	imaginary unit
k	summation index
p	variable defined by Eq. (*15*)
q	variable defined by Eq. (*15*)
Q	smoothing effect of mixer
$r(\tau)$	normalized autocorrelation function
$R(\tau)$	autocorrelation function
t	residence time (T)
u	auxiliary variable (T)
v	auxiliary variable (T)
$X(t)$	random input signal of mixer
$Y(t)$	random response
β	parameter of gamma-distribution (*3*)
$\Gamma(a)$	gamma-function
$\varkappa$	parameter of gamma-distribution (*3*) (T^{-1})
ϱ	variable defined by Eq. (*12*)
σ^2	variance
τ	time (T)
φ	variable defined by Eq. (*13*)

Subscripts

c critical value
i ideal mixer
m maximum value
o optimum value
p piston flow
x input signal
y response

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